

# Quark deconfinement in neutron star cores and the ground state of neutral matter

Chang-Qun Ma and Chun-Yuan Gao\*

School of Physics, Peking University, Beijing 100871, China

February 1, 2008

## Abstract

Whether or not deconfined quark phase exists in neutron star cores and represents the ground state of neutral matter at moderate densities are open questions. We use two realistic effective quark models, the three-flavor Nambu-Jona-Lasinio model and the modified quark-meson coupling model, to describe the neutron star matter. After constructing possible hybrid equations of state (EOSes) with unpaired or color superconducting quark phase, we systematically discuss the observational constraints of neutron stars on the EOSes. It is found that the neutron star with pure quark matter core is unstable and the hadronic phase with hyperons is denied, while hybrid EOSes with two-flavor color superconducting phase or unpaired quark matter phase are both allowed by the tight and most reliable constraints from two stars Ter 5 I and EXO 0748-676. And the hybrid EOS with unpaired quark matter phase is allowed even compared with the tightest constraint from the most massive pulsar star PSR J0751+1807. Therefore, we conclude that the ground state of neutral matter at moderate densities is in deconfined quark phase likely.

PACS number(s): 12.38.Mh, 12.39.-x, 26.60.+c

## 1 INTRODUCTION

Neutron stars are some of the densest objects in the universe and the density in the inner core of a neutron star could be as large as several times nuclear saturation density ( $\cong 0.17 \text{ fm}^{-3}$ ) [1]. The core of neutron star is so dense that phase transition from confined hadronic phase to deconfined quark phase may exist. The possible emergence of deconfinement phase in neutron star cores has aroused great interests since it may have a distinct effect on the neutron star structure[2]. Up to now, however, no conclusive observational or experimental evidence suggests that the quark matter core conjecture is true and this still

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\*Electronic address: gaocy@pku.edu.cn

remains an open question. In our recent work where the hadronic phase containing octet baryons was considered, it was found that the equation of state (EOS) with deconfinement phase would be ruled out by the observational mass limit of  $1.68M_{\odot}$  of star Ter 5 I[3]. Similar result was also deduced by Özel who reported that EOSes with exotic phases would be ruled out by the inferred mass and radius for star EXO 0748-676 and she concluded that the ground state of matter was hadrons and not deconfined quarks[4]. While Alford et al. compared Özel's observational limits with predictions based on a more comprehensive set of proposed equations of state from the literature, and concluded that the presence of quark matter in EXO 0748-676 was not ruled out[5]. Therefore, the existence of deconfined quark phase in neutron star cores and the ground state of neutral matter at moderate densities are controversial. Our goal of the present paper is to systematically investigate the observational constraints on the deconfinement in neutron star cores and check whether deconfined quarks are possible to exist in the ground state of neutral matter at moderate densities.

It is expected that at extreme conditions chiral symmetry can be restored and quarks and gluons become deconfined[6]. Consequently, quark matter was frequently dealt with as a non-interacting quark gas, i.e., unpaired quark matter (UQM), and usually described by a phenomenological bag model[7]. According to the BCS theory[8], any attractive interaction in a cold fermi sea will cause Cooper instability in the vicinity of fermi surface in the momentum space and superconductor will be formed. Because of the attractive quark-quark interaction in the color antitriple channel[9] it is expected that color superconducting state, with a spontaneous breakdown of the non-Abelian SU(3) color gauge group, would be the ground state of quark matter. The color superconducting state has attracted great interests since it was found that superconducting gap could be  $\sim 100\text{MeV}$  due to the nonperturbative features[10]. Depending on quarks participating in a diquark condensation, one can distinguish several color superconducting phases. The prominent two are the two-flavor color superconducting (2SC) phase containing  $ud$  pairs together with unpaired strange quarks and the color-flavor locked (CFL) phase containing  $ud$ ,  $ds$  and  $us$  pairs (for recent reviews see Ref. [11]).

We adopt two realistic effective quark models for quark and hadronic phases respectively to describe the neutron star matter. The extended three-flavor Nambu-Jona-Lasinio (NJL) model with determinant interaction[12], which shares many symmetries with QCD, is used to calculate the properties of quark matter in UQM, 2SC and CFL phases. For hadronic phase, we adopt the EOSes of neutron-proton matter and hypernuclear matter with octet baryons by the improved modified quark-meson coupling (MQMC) model[3, 13]. The MQMC model gives satisfactory description for saturation properties of nuclear matter[14] and could reproduce the bulk properties of finite nuclei well[15]. We then assume a sharp (first order) transition from pure hadronic to pure quark phase and consider only the homogeneous quark phase and not mixed phase between different quark phases.

The outline of this paper is as follows: In Sec. 2, we briefly introduce the three-flavor NJL model for dense quark matter and the MQMC model for

hadronic phase. In Sec. 3, we construct the possible equations of state of neutron star matter and then discuss the observational constraints on the EOSes. Sec. 4 is devoted to summaries and conclusions.

## 2 THE MODEL

### 2.1 QUARK PHASE

Strange quark matter at moderate densities can be effectively described by the three-flavor NJL model[16, 17]. The Lagrange density of extended three-flavor NJL model with six-fermion determinant interaction (t'Hooft term) is given by

$$\mathcal{L}_{\text{NJL}} = \bar{q} (i\gamma^\mu \partial_\mu - \hat{m}_0) q + \mathcal{L}_{\bar{q}q} + \mathcal{L}_{qq}, \quad (1)$$

where

$$\begin{aligned} \mathcal{L}_{\bar{q}q} = & G \sum_{a=0}^8 \left[ (\bar{q} \tau_a q)^2 + (\bar{q} i\gamma_5 \tau_a q)^2 \right] \\ & - K [\det_f (\bar{q}(1 + \gamma_5)q) + \det_f (\bar{q}(1 - \gamma_5)q)] \end{aligned} \quad (2)$$

and

$$\mathcal{L}_{qq} = H \sum_{A=2,5,7} \sum_{A'=2,5,7} (\bar{q} i\gamma_5 \tau_A \lambda_{A'} q^c) (\bar{q}^c i\gamma_5 \tau_{A'} \lambda_A q). \quad (3)$$

Here,  $q = (u, d, s)^T$  denotes the quark fields with three colors. And the current quark mass matrix has the form  $\hat{m}_0 = \text{diag}(m_{0u}, m_{0d}, m_{0s})$  in the flavor space, where  $m_{0u} = m_{0d} = m_{0q}$  is assumed throughout this paper.  $\tau_0 = \sqrt{\frac{2}{3}}\mathbb{1}$  is proportional to the unit matrix in the flavor space.  $\tau_A$  and  $\lambda_A$  ( $A=1, \dots, 8$ ) are Gell-Mann matrixes in flavor and color spaces respectively.  $q^c = C\bar{q}^T$  is the charge-conjugate spinor.

In the present work, we restrict ourselves to bulk quark matter in mean-field approximation and focus on the chiral condensates defined as

$$\phi_f = \langle \bar{q}_f q_f \rangle, \quad f = u, d, s, \quad (4)$$

and the three-flavor diquark condensates being

$$\Delta_A = -2H \langle \bar{q}^c \gamma_5 \tau_A \lambda_A q \rangle, \quad A = 2, 5, 7. \quad (5)$$

After bosonization, one obtains the linearized version of the model in the mean-field approximation,

$$\begin{aligned} \mathcal{L} = & \bar{q} (i\gamma^\mu \partial_\mu - \hat{m}) q \\ & + \frac{1}{2} \sum_A [\bar{q} (\Delta_A \gamma_5 \tau_A \lambda_A) q^c + \bar{q}^c (-\Delta_A^* \gamma_5 \tau_A \lambda_A) q] \\ & - \frac{1}{4H} \sum_A |\Delta_A|^2 - 2G \sum_f \phi_f^2 + 4K \phi_u \phi_d \phi_s, \end{aligned} \quad (6)$$

where we have introduced the constituent quark mass

$$\hat{m} = \begin{pmatrix} m_{0u} - 4G\phi_u + 2K\phi_d\phi_s & & \\ & m_{0d} - 4G\phi_d - 2K\phi_s\phi_u & \\ & & m_{0s} - 4G\phi_s - 2K\phi_u\phi_d \end{pmatrix}. \quad (7)$$

Employing Nambu-Gorkov formalism, then the thermodynamic potential per unit volume at temperature  $T$  is obtained via the finite temperature field theory[18] and takes the form,

$$\Omega = -\frac{T}{2V} \sum_{\vec{P}} \sum_{i=1}^{72} \left[ \frac{|\omega_i|}{2T} + \ln \left( 1 + e^{-|\omega_i|/T} \right) \right] + \Omega_{\text{const}} + \Omega_e, \quad (8)$$

where

$$\Omega_{\text{const}} = 2G \sum_{f=u,d,s} \phi_f^2 - 4K\phi_u\phi_d\phi_s + \frac{1}{4H} \sum_A |\Delta_A|^2, \quad (9)$$

and

$$\Omega_e = -\frac{1}{12\pi^2} \left( \mu_e^4 + 2\pi^2 T^2 \mu_e^2 + \frac{7\pi^4}{15} T^4 \right) \quad (10)$$

is the contribution from the electron gas with chemical potential  $\mu_e$ .

In Eq.(8),  $\omega_i$  is the energy of a quasiparticle and can be obtained by diagonalizing the inverse propagator in Nambu-Gorkov basis. Or, equivalently,  $\omega_i$  can be obtained by calculating the eigenvalues of the  $72 \times 72$  matrix

$$\mathcal{M} = \begin{bmatrix} \vec{P} \cdot \vec{\alpha} + \hat{m}\gamma_0 - \hat{\mu} & -\sum_A \Delta_A \gamma_0 \gamma_5 \tau_A \lambda_A \\ \sum_A \Delta_A^* \gamma_0 \gamma_5 \tau_A \lambda_A & \vec{P} \cdot \vec{\alpha} + \hat{m}\gamma_0 + \hat{\mu} \end{bmatrix}. \quad (11)$$

And the chemical potential operator  $\hat{\mu}$  is a diagonal  $9 \times 9$  matrix in flavor and color space. By introducing the quark number chemical potential  $\mu$ , electrochemical potential  $\mu_Q$  and two additional chemical potentials  $\mu_3$  and  $\mu_8$  coupled to the color charges  $\lambda_3$  and  $\lambda_8$  respectively,  $\hat{\mu}$  can be expressed as (see Ref. [16] for details)

$$\hat{\mu} = \mu + \mu_Q \left( \frac{1}{2} \tau_3 + \frac{1}{2\sqrt{3}} \tau_8 \right) + \mu_3 \lambda_3 + \mu_8 \lambda_8. \quad (12)$$

In beta equilibrium, we have

$$\mu_e = -\mu_Q. \quad (13)$$

The order parameters,  $\phi_f$  and  $\Delta_A$ , can be obtained by minimizing the thermodynamic potential, and are equivalently given by the gap equations,

$$\frac{\partial \Omega}{\partial \phi_f} = 0, \quad f = u, d, s, \quad (14)$$

$$\frac{\partial \Omega}{\partial \Delta_A} = 0, \quad A = 2, 5, 7. \quad (15)$$

With the diquark condensates, we can explicitly distinguish different quark phases as:

$$\begin{aligned}\Delta_2 = \Delta_5 = \Delta_7 = 0 : & \quad \text{UQM;} \\ \Delta_5 = \Delta_7 = 0, \Delta_2 \neq 0 : & \quad \text{2SC;} \\ \Delta_2, \Delta_5, \Delta_7 \neq 0 : & \quad \text{CFL.}\end{aligned}$$

Dense quark matter in neutron star is electrical and color neutral, then the following relations should be maintained:

$$n_Q = -\frac{\partial \Omega}{\partial \mu_Q} = 0, \quad (16)$$

$$n_3 = -\frac{\partial \Omega}{\partial \mu_3} = 0, \quad (17)$$

$$n_8 = -\frac{\partial \Omega}{\partial \mu_8} = 0. \quad (18)$$

Finally, for a given quark number density

$$n = -\frac{\partial \Omega}{\partial \mu},$$

the energy density  $\mathcal{E}$  and pressure  $\mathcal{P}$  at zero temperature are given by:

$$\mathcal{E} = \Omega + \Omega_{\text{vac}} + \mu n, \quad (19)$$

$$\mathcal{P} = -\Omega - \Omega_{\text{vac}}. \quad (20)$$

$\Omega_{\text{vac}}$  is chosen so that  $\mathcal{P}$  and  $\mathcal{E}$  vanish in vacuum, which is the only way to uniquely determine the EOS within the NJL model without any further assumption.

For the model parameters, we take the values as follows. To regularize the divergent integrals we need a sharp cutoff  $\Lambda$  in 3-momentum space since the NJL model is nonrenormalizable. Thus we have a total of 6 parameters, namely, the current masses  $m_{0s}$  and  $m_{0q}$  for strange and nonstrange quarks, the three couplings  $G$ ,  $K$  and  $H$ , and the cutoff  $\Lambda$ . Following the method adopted in Ref.[19], we get  $\Lambda=602.8\text{MeV}$ ,  $G\Lambda^2=1.803$ ,  $K\Lambda^5=12.93$  and  $m_{0s}=140.9\text{MeV}$  by fitting the meson masses[20]  $m_\pi = 134.98\text{MeV}$ ,  $m_K = 497.65\text{MeV}$  and  $m_{\eta'} = 957.78\text{MeV}$  and the  $\pi$  decay constant  $f_\pi = 92.2\text{MeV}$ [21] while  $m_{0q}$  is fixed at  $5.5\text{MeV}$ .  $H = G$  is set as has been used in Ref.[16].

## 2.2 HADRONIC PHASE

For the hadronic phases, we focus on the neutron-proton matter and the hypernuclear matter consisting of the baryon octet, i.e., p, n,  $\Lambda$ ,  $\Sigma^+$ ,  $\Sigma^0$ ,  $\Sigma^-$ ,  $\Xi^0$  and  $\Xi^-$ . The hadronic phases are described with the improved MQMC model which has been discussed in detail in Ref.[3]. In order to make the calculation for hadronic phases compatible with that for quark phases, we should take the

Table 1: The new zero-point motion parameters and radii for the modified quark-meson coupling model.

	M(MeV)	Z	R(fm)
N	939.0	2.0403	0.6000
$\Lambda$	1115.7	1.8099	0.6459
$\Sigma^+$	1189.4	1.6318	0.6722
$\Sigma^0$	1192.6	1.6236	0.6733
$\Sigma^-$	1197.4	1.6114	0.6749
$\Xi^0$	1314.8	1.4743	0.6922
$\Xi^-$	1321.3	1.4567	0.6942

Table 2: New independent coupling constants, which have been defined in Ref.[3].

$g_{\sigma}^{u,d}$	$g_{\omega}^{u,d}$	$g_{\rho}^{u,d}$	$g_{\sigma}^{\text{bag},N}$
0.9685	2.7071	7.9288	6.8732

same current quark masses in both models. Therefore the current quark masses take  $m_{0u} = m_{0d} = 5.5\text{MeV}$  and  $m_{0s}=140.9\text{MeV}$ , which are different from those used in Ref.[3], and the other inputs, such as the mass spectrum and the nucleon's radius, are unchanged. As a consequence, the bag constant in vacuum is changed to be  $B_0^{1/4}=187.7716\text{MeV}$ . The obtained zero-point motion parameters and bag-radii for baryons and coupling constants are all changed, their new values are listed in TABLE 1. and TABLE 2, respectively.

### 3 RESULTS AND DISCUSSIONS

#### 3.1 EQUATION OF STATE

First order phase transition takes place at critical point where baryon chemical potentials and pressures for the two phases are equal. The pressures of hadronic and quark phases are shown in Figure 1 as a function of baryon chemical potential and the position of hadron-quark phase transition can be easily read off as the point where the lines  $\mathcal{P}(\mu_B)$  cross. From the figure, we can find four possible hadron-quark phase transitions, i.e., A, B, C and D. EOS of the hypernuclear matter(npH) has no cross with those of quark phases 2SC or UQM, and has only one cross at A with that of CFL. Whereas the neutron-proton matter(np) could change into deconfined quark phases in UQM, 2SC or CFL state at B, C and D. Therefore, we could construct totally four kinds of hybrid EOSes between hadronic phase and quark deconfinement phase.

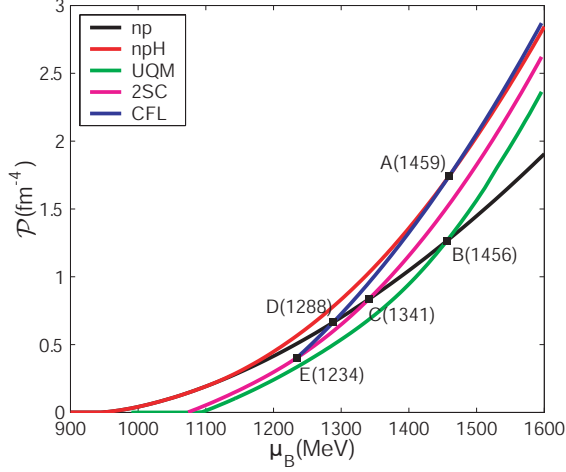


Figure 1: Pressure of hadronic or quark phase as a function of baryon chemical potential. np (black) denotes neutron-proton matter and npH (red) denotes the hypernuclear matter consisting of octet baryons. UQM (green), 2SC (magenta) and CFL (blue) denote the three kinds of homogeneous quark phases. Squares ordered by A, B, C and D mark the hadron-quark phase transitions and E is the phase transition between 2SC and CFL phases. Values in parentheses are the critical chemical potentials.

EOSes are plotted in Figure 2 and possible phase transitions are also shown by dotted lines marked by A, B, C, D and E. The hybrid EOS of npH+CFL can be constructed by combining the EOS of pure hypernuclear matter npH and the EOS of pure quark matter in CFL state for low and high pressures respectively, with a hadron-quark phase transition at A. The other EOSes of np+UQM, np+2SC and np+CFL could be similarly constructed. There are some notable features for the hybrid EOSes. First, the hadron-quark phase transitions are first order, which is an inevitable result because we have chosen a sharp interface as claimed in Sec. 1. Second, the energy gaps of phase transitions are rather large, which is found to have profound effect on the star structure and is going to be discussed. Among the hybrid EOSes, the stiffest one is of np+UQM, and the next are of np+2SC, np+CFL and npH+CFL in turn.

### 3.2 MAXIMUM MASS

By solving the Tolman-Oppenheimer-Volkoff equations [22], mass-radius relations are obtained for different equations of state, which are given in Figure 3. The softest EOS of npH+CFL predicts a maximum mass of  $1.54 M_{\odot}$ . Compared with the best measured pulsar mass  $1.44 M_{\odot}$  in the binary pulsar PSR 1913+16[23], which had been taken as the lower limit of neutron star's maximum mass for many years, all the EOSes here are allowed. However, very recent

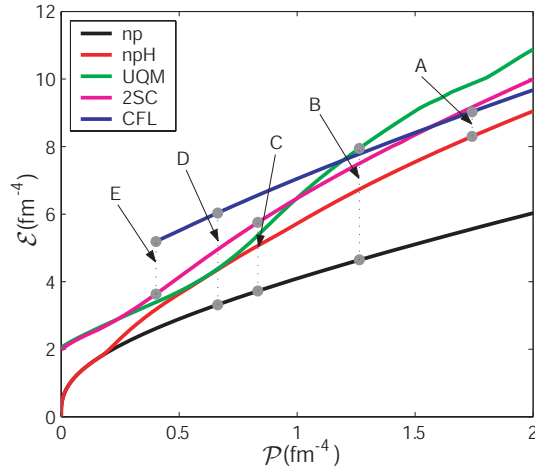


Figure 2: Energy vs. pressure for equations of state. The solid lines represent the EOSes of pure phases. And the thin dotted lines indicate the first order phase transitions. A, B, C, D and E are the same as those defined in Figure 1.

measurements strongly indicate that there are more massive neutron stars. The typical neutron star is reported by Ransom et al. who inferred that at least one of the stars in Terzan 5, the Ter 5 I, is more massive than  $1.48$ ,  $1.68$ , or  $1.74 M_{\odot}$  at 99%, 95%, and 90% confidence levels[24]. Therefore, as indicated in Figure 3, imposing the tighter observational constraint of  $1.68 M_{\odot}$  at 95% confidence level, pure hadronic EOS with hyperons and the hybrid EOS of npH+CFL are firmly ruled out, while the hybrid EOSes of np+UQM, np+2SC and np+CFL can be compatible with this constraint. So far, the most massive pulsar star reported is PSR J0751+1807, which has a inferred mass of  $2.1 \pm 0.2 M_{\odot}$  covering 68% confidence uncertainties[25]. Therefore, PSR J0751+1807 gives the tightest mass constraint of  $1.9 M_{\odot}$  at about 68% confidence level. If this tightest mass limit is confirmed, all the hybrid EOSes are ruled out but that of np+UQM. And, of course, the EOS of pure neutron-proton matter is allowed because it is stiffer than the one of np+UQM. However, the measurement uncertainty of PSR J0751+1807 is not yet small enough to draw any firm conclusion and the tight constraint coming from Ter 5 I is more reliable.

The hybrid stars with quark phases are indicated in Figure 3 with dashed lines, which reveal that all the stars with quark phase cores are unstable, i.e., those with quark cores will collapse. The direct reason is that hybrid EOS has a rather large discontinuity of energy caused by the first order phase transition as claimed in Sec. 3.1. Our result confirms those obtained in earlier works, where the hadronic phase has been described by different models, e.g., the relativistic mean field model or the microscopic many-body theory[26, 27]. Nevertheless Buballa et al. reported recently that by adopting another parameter set which was determined by a different method from here, 2SC phase was possible to



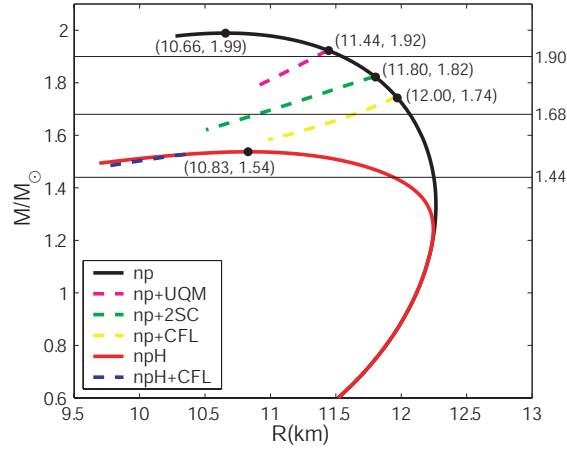


Figure 3: Neutron star mass-radius relations for pure or hybrid EOSes. Solid lines indicate the pure hadronic star and dashed lines are for hybrid stars. The dots denote the maximum masses and their coordinates are given in the parentheses. Three horizontal lines represent the observational constraints from stars PSR 1913+16 ( $1.44M_{\odot}$ [23]), Terzan 5 I ( $1.68M_{\odot}$ [24]) and PSR J0751+1807 ( $1.90M_{\odot}$ [25]) respectively.

exist in the star core within a very tiny window[28]. Baldo et al. suggested that the instability may be linked to the lack of confinement in the current NJL model[27]. Moreover, parameters obtained by fitting the vacuum properties might not be suitable at high densities. Therefore if the existence of pure quark cores in neutron stars was confirmed by future observations, the NJL model currently used should be modified.

### 3.3 GRAVITATIONAL REDSHIFT

In principle, the range of gravitational redshift predicted by an EOS should cover all the ever detected redshifts. Cottam et al. have reported a redshift  $z$  of 0.35 inferred by identifying three sets of transitions in the spectra of x-ray binary EXO 0748-676[29]. The result has been confirmed by Chang et al.[30]. And it has been shown that the total error in this redshift is no more than 5%[31]. Therefore, allowing for the error bar, it imposes a lower limit of about 0.3325 to the maximum redshift. In Figure 4, gravitational redshifts vs. masses of different EOSes are shown. EOSes with hyperons fail to construct stable neutron star satisfying the redshift of EXO 0748-676, so they are ruled out. Without hyperons, hybrid EOS of np+CFL is not allowed likely, while those stiffer, namely, of np+2SC, np+UQM and np, are permitted.

Larger observational redshift gives more stringent constraint. Tiengo, et al., inferred that a redshift  $z=0.4$  can be obtained by explaining the emission lines from 4U1700+24 with the Ne IX triplet[32]. Then the hybrid EOS of

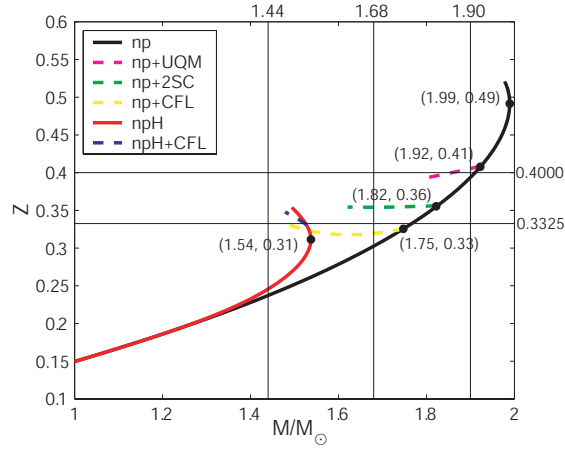


Figure 4: Gravitational redshifts vs. masses for different EOSes. The lower horizontal line is the observational gravitational redshift of EOX0748-676[29, 31] and the upper one is that of 4U1700+24[32]. The dots denote the maximum masses and their coordinates are given in the parentheses. Constraints of observational masses (the same as in Figure 3) are also plotted with three vertical lines.

np+2SC would be ruled out by this constraint, and only the one of np+UQM is marginally permitted. However, due to the lack of identifications of other spectral features at  $z=0.4$ , it seems that the interpretation of  $z=0.012$  for 4U1700+24 is more reliable[32]. Therefore redshift of EXO 0748-676 is the most reliable constraint till now.

Combining the constraints of observational masses and redshifts of neutron stars, it can be summarized that EOSes of np+2SC, np+UQM and np are very likely to be favored. And if either the mass limit of  $1.9M_{\odot}$  of the most massive pulsar star PSR J0751+1807 or the very tentative interpretation of redshift of 4U1700+24 equal to 0.4 is confirmed, EOSes with CFL or 2SC quark phase are both denied, then the allowed hybrid EOS is only of np+UQM. It should be noted that rotation effect is not considered here because influences of rotation effect on static properties are negligible for the main observations[33] we currently discussed.

## 4 SUMMARIES AND CONCLUSIONS

We have used two realistic effective quark models, i.e., the three-flavor NJL model and the MQMC model, to describe the neutron star matter. For hadronic phase, EOSes with and without hyperons (npH and np) are both considered. And we have discussed the quark matter phase in normal and color superconducting states, namely, UQM, 2SC and CFL. Then four possible hybrid EOSes between hadronic phase and quark deconfinement phase are constructed. We

find that EOSes with hyperons should be ruled out by the observational constraints from the mass of star Ter 5 I and/or the redshift of binary star EXO 0748-676. Moreover, the hybrid EOS of np+CFL is also ruled out by the observational redshift of EXO 0748-676. As a consequence, hybrid EOSes of np+2SC and np+UQM as well as pure hadronic EOS of np are most likely to be favored. Tightest but less reliable constraint can be inferred by the mass of the most massive pulsar star PSR J0751+1807, and if it is confirmed the permitted hybrid EOS is of np+UQM only.

Therefore, we conclude that observational constraints of neutron star could not rule out all the possible EOSes with quark phase though the neutron stars with pure quark cores are found to be unstable in our calculation. Both normal unpaired quark state and two-flavor color superconducting state are likely permitted, and future observations are needed to determine which of them (or none) is the right ground state at this density. So the ground state of neutral matter at moderate densities could be in deconfined quark phase.

## ACKNOWLEDGMENTS

Financial support by the National Natural Science Foundation of China under grants 10305001, 10475002 & 10435080 is gratefully acknowledged.

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